

INTEGRATED OPTIMIZATION OF NONLINEAR R/C FRAMES
WITH RELIABILITY CONSTRAINTS

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Summary

A structural optimization algorithm was researched including global displacements as decision variables. The algorithm was applied to planar reinforced concrete frames with nonlinear material behavior submitted to static loading. The flexural performance of the elements was evaluated as a function of the actual stress-strain diagrams of the materials. Formation of rotational hinges with strain hardening were allowed and the equilibrium constraints were updated accordingly. The adequacy of the frames was guaranteed by imposing as constraints required reliability indices for the members, maximum global displacements for the structure and a maximum system probability of failure.

Previous Research

Structural frame optimization problems have been usually formulated based on the cycling between two distinct phases: analysis and optimal design. The option described in this work combines both phases by the addition of the global displacements to the set of design variables, option researched by several authors (ref. 1 and 2). The main purpose of this strategy was to determine the benefits of extending the linear static formulation to nonlinear static structural problems using the secant stiffness method. The reason behind this research was the fact that the global stiffness changes created by the nonlinear behavior would be considered simultaneously with the changes of the element sizes, thus improving convergence.

The first step of the research was to optimize elastic plane frames with elements with rectangular sections submitted to static loading. The objective function was the volume of the structure and constraints of the optimization problem were equalities representing global equilibrium of the structure and inequalities for the limits on global displacements and the maximum flexural stresses. The strategy adopted consisted of transforming the constrained problem in an unconstrained one using the method of the Augmented Lagrangian Multipliers (ref. 3). The unconstrained minimization was solved using the Hooke and Jeeves method.

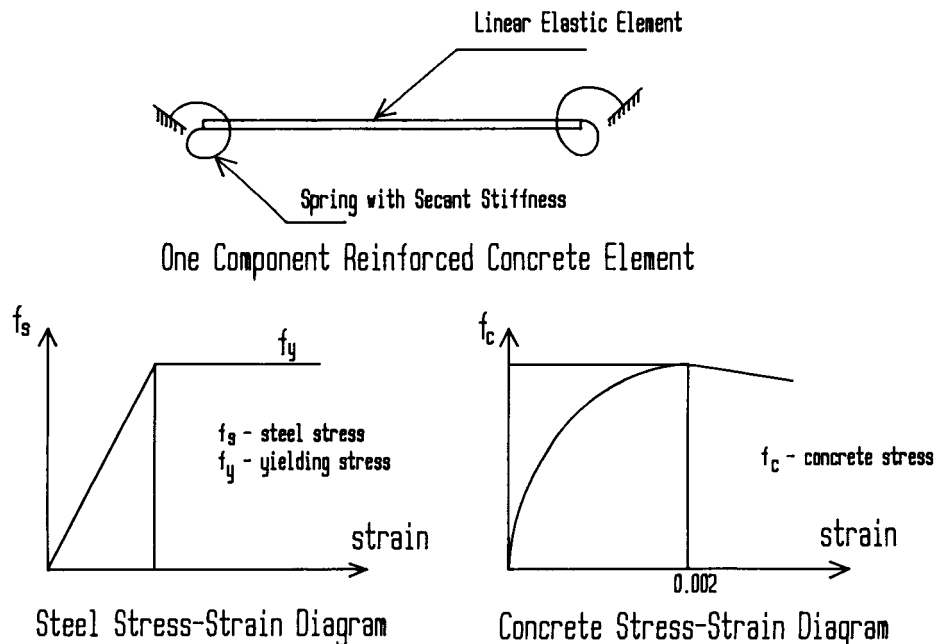
The results with this formulation were encouraging and the optimal solutions were found. The convergence rate was dependent on the initial design, scaling, penalty parameters and lagrangian multipliers values. The computational effort was considerable when compared with other explored techniques based on optimality criteria and mathematical programming methods. To improve the efficiency of the algorithm a gradient technique was implemented to solve the unconstrained minimization problem. This improvement was unsuccessful since the Augmented Lagrangian function was very steep, with large sensitivity to any small variation of the displacement variables.

Nonlinear Formulation

The following logical objective was to extend this strategy to nonlinear reinforced concrete frames. The typical frame element has rectangular cross section and is doubly reinforced with equal amount of flexural steel on both sides. The model adopted for the inelastic reinforced concrete element was the one component model, where rotational springs are added to the ends of the elastic element to simulate the formation of plastic hinges at the extremities of the element (ref. 4). The stiffnesses of the linear elastic element stiffness and the springs was condensed using the flexibility formulation.

The determination of the characteristics of each reinforced concrete section was based on the stress strain diagrams for the concrete and the reinforcing steel. The yielding and ultimate moments for each cross section were used to determine the characteristics of the springs for each element. The spring stiffness was considered infinite whenever the element moment was below the yielding moment. When the moment was above the yielding value the spring stiffness was updated and, since there was no incremental loading or unbalanced iteration of the structure, the secant stiffness was adopted for the spring stiffness (ref. 5).

ONE COMPONENT MODEL AND MATERIAL CHARACTERISTICS



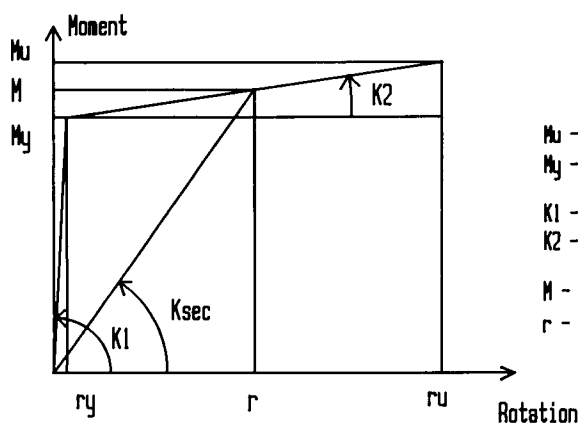
Procedure Implementation

The objective function was the cost of the materials: concrete and steel. The constraints included the equilibrium equalities, maximum global displacements and maximum probability of failure for each element. The equilibrium constraints were evaluated every time a design variable changed with the corresponding updating of the spring stiffnesses. The values of the maximum displacements were dictated by serviceability constraints like maximum joint rotations or story drifts. The maximum element probabilities of failure were chosen with current practices of structural design codes (ref. 6).

Flexural element actions are the most important in small and medium sized frames for the definition of section sizes and longitudinal steel. The evaluation of element reliability was based on the corresponding flexural failure function (ref. 7). The same approach was used to evaluate the system probability of failure. The basic variables considered were the compressive strength of concrete and the external loads.

The system probability of failure was evaluated at the mechanism level at the end of each optimization cycle. If the value of the system probability of failure was not satisfactory the optimization was restarted using a different limit of element probability of failure for the elements involved in the failure mechanism. The system probability of failure was obtained using the beta-unzipping method (ref. 8). In summary, the elementary mechanisms of failure were determined using Watwood's method (ref. 9) and the correspondent failure functions formed. These mechanisms were then combined linearly and the related probabilities of failure calculated, while rejecting those combinations with values outside given intervals.

SPRING SECANT STIFFNESS



Spring Moment-Rotation Diagram

M_u - Ultimate moment
 M_y - Yielding moment

r_u - Ultimate rotation
 r_y - Yielding rotation

K_1 - $10e30$
 K_2 - $(M_u - M_y) / (r_u - r_y)$

K_{sec} - Spring stiffness for
 $M > M_y$

M - actual element moment
 r - actual element chord rotation

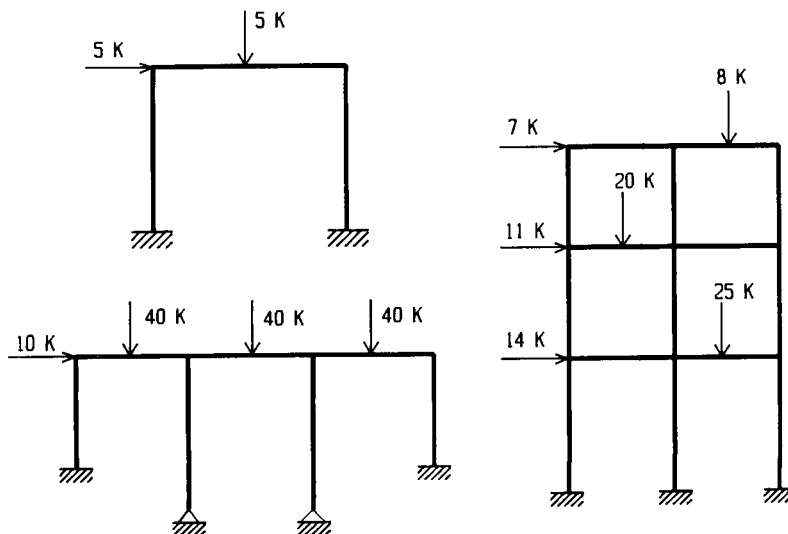
Optimization Results

The first approach to solve the optimization problem used the Augmented Lagrangian function and the Hooke and Jeeves method as the unconstrained minimization technique. Considerable effort was put into this formulation with several options for the starting points, combinations of penalty parameters, scaling techniques and number of cycles. Three structures were tested with different levels of complexity.

In some cases the values obtained were close with those corresponding to the expected optimal values. Reliability constraints were satisfied, displacements were within the limits and equality constraints were satisfied. However, convergence was difficult to obtain and largely dependent on several different choices made at the start of the optimization cycle. At the same time the element forces were not in accordance with the assumed secant spring stiffnesses showing lack of convergence of the nonlinear iteration process.

To improve convergence of the nonlinear equilibrium of the structure an intermediate phase was created in the optimization process. The displacements were removed from the optimization cycle. This phase corresponded to the solution of the equilibrium equations every time a cycle of variable optimization in the Hooke and Jeeves was completed. The displacements were obtained from the equilibrium equations assuming the values of the secant spring stiffnesses as those at the end of the cycling optimization. The results were nevertheless the same as before and for that reason another technique was implemented.

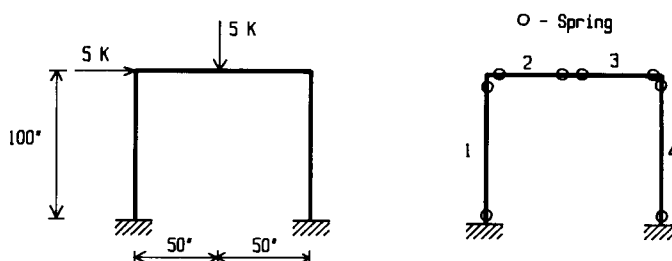
EXAMPLES TESTED



The Generalized Reduced Gradient method was chosen because of its characteristic of solving iteratively a set of nonlinear equations. The algorithm used (ref. 10) performed very well for the elastic case with convergence in most of the cases. It proved to be almost insensitive to the initial design points. The extension to the nonlinear material behavior is however in process. A first phase of this extension consisted of assuming for the secant spring stiffness a yielding value, i.e., corresponding to the ratio of the yielding moment and the yielding rotation whenever the element moment was greater than the yielding moment. Equilibrium constraints were satisfied, element moments were in accordance with the assumed spring stiffnesses and the global displacements were in accordance with assumed spring stiffness values.

The second phase of transforming the spring stiffness from the yielding stiffness to the secant stiffness is presently being researched. The initial design for the values presented using the secant spring stiffness formulation was obtained with the elastic stiffness version having as ultimate moment the yielding moment. The secant stiffness values were limited to a minimum value corresponding to the ratio of the ultimate moment and the ultimate rotation to prevent severe oscillations of these values. The results verified the equilibrium constraints within certain tolerance, the global displacements were those corresponding to the element spring stiffnesses, reliability constraints were satisfied and there was an improvement of the cost of the structure. All elements have yielded corresponding to the expected results from a optimal configuration. However, there are discrepancies in the moments at the joints which shows a insufficient convergence of the equilibrium constraints.

RESULTS WITH YIELDING STIFFNESS AND SECANT STIFFNESS



ELEMENT	INITIAL			ELASTIC			YIELDING			SECANT		
	b	h	As	b	h	As	b	h	As	b	h	As
1	5.0	10.0	1.0	2.0*	6.0*	.20	2.0*	6.0*	.21	2.0*	6.0*	.38
2	5.0	10.0	1.0	2.0*	6.0*	.23	2.0*	6.0*	.21	2.0*	6.0*	.25
3	5.0	10.0	1.0	2.0*	11.1	.73	2.0*	9.75	.64	2.0*	9.81	.64
4	5.0	10.0	1.0	2.0*	12.4	.81	2.0*	10.9	.71	2.0*	10.9	.72
Cost	18,000			6,890			6,296			6,505		

b - base (in); h - height (in); As - steel area (in²); * - lower bounds.

Directions Headed

The Augmented Lagrangian function together with the Hooke and Jeeves for the unconstrained minimization although insensitive to discontinuities of the constraints is not the best choice when compared with the Generalized Reduced Gradient method. More research has to be done on the robustness of the convergence of the nonlinear iteration process (ref. 11).

A possible improvement would consist of a similar enhancement to the one applied in the Augmented Lagrangian formulation with an intermediate solution of the displacements during the optimization cycle. Another possible improvement would be the use of a cycling procedure where the spring stiffness values were kept constant during the optimization and updated at the end with consequent optimization cycle until there was stabilization of the spring stiffness values.

The integrated approach proved itself adequate for the elastic stiffness if an adequate mathematical programming technique is chosen. It is logical to expect that it will probably perform well in the case of inelastic stiffness if adequately integrated with some kind of nonlinear structural analysis technique.

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